National Centre for Excellence in the Teaching of Mathematics

MathsHUBS

Big Ideas in Mastery: Coherence

Messages

1. Small steps are easier to take.
2. Focussing on one key point each lesson allows for deep and sustainable learning.
3. Certain images, techniques and concepts are important pre-cursors to later ideas. Getting the sequencing of these right is an important skill in planning and teaching for mastery.
4. When something has been deeply understood and mastered, it can and should be used in the next steps of learning.

For example:

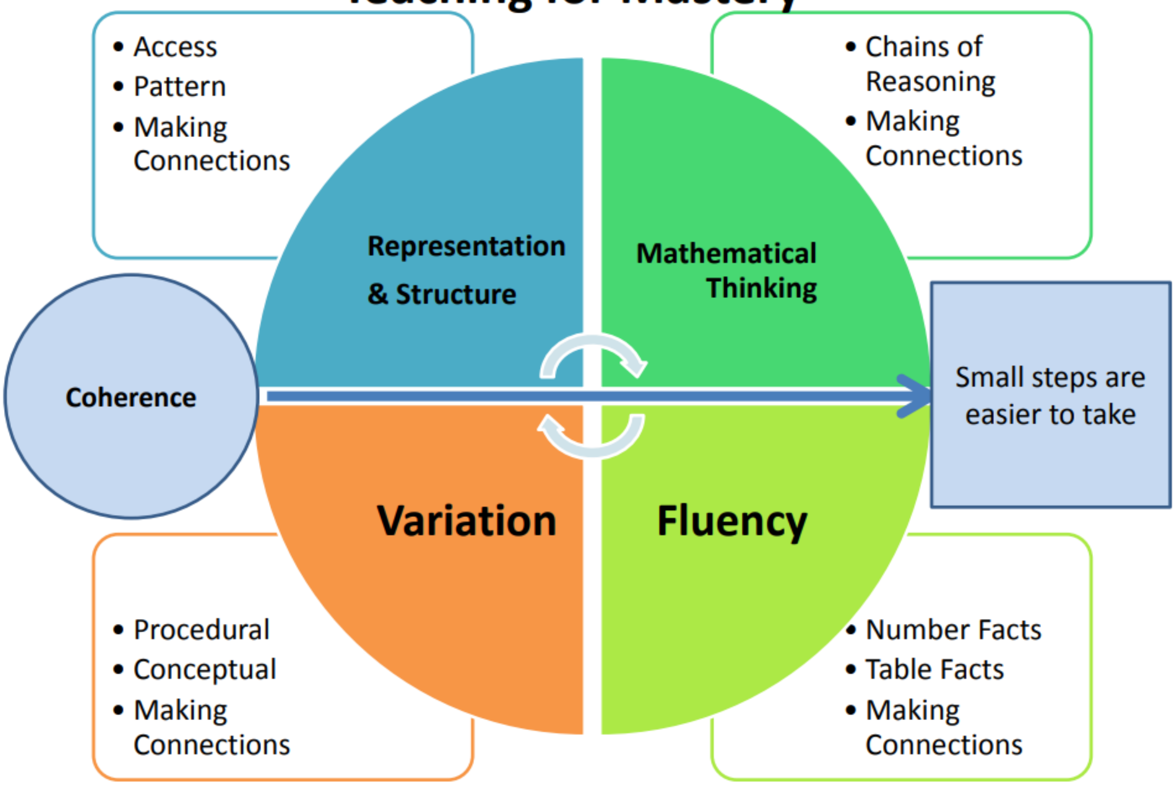
Before teaching the written algorithm for subtraction:

$$\begin{array}{r} 47 \\ \underline{-\ 38} \end{array}$$

Pupils need to:

- be fluent in their number facts for single digit numbers
- have a good understanding that 47 can be partitioned into 40 and 7 or 30 and 17
- understand that 40 can be thought of as 4 tens
- understand that 3 tens and 4 tens make 7 tens and that this is the same as 30 and 40 make 70.

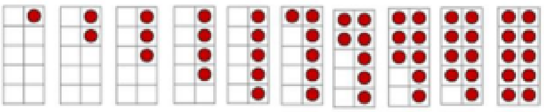
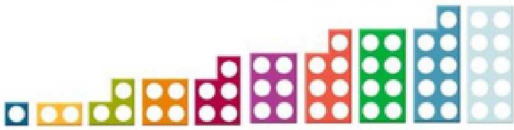
Teaching for Mastery



Big Ideas in Mastery: Representation & Structure

Messages
1. The representation needs to pull out the concept being taught, and in particular the key difficulty point. It exposes the structure.
2. In the end, the children need to be able to do the maths without the representation
3. A stem sentence describes the representation and helps the children move to working in the abstract ("ten tenths is equivalent to one whole") and could be seen as a representation in itself
4. There will be some key representations which the children will meet time and again
5. Pattern and structure are related but different: Children may have seen a pattern without understanding the structure which causes that pattern

For example:
Here are two representations for numbers within 10; the tens frame and Numicon:

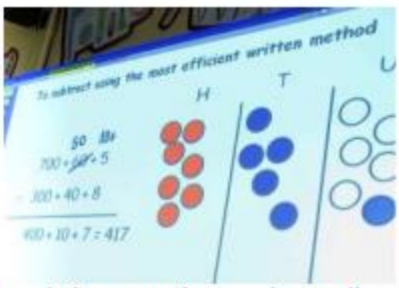



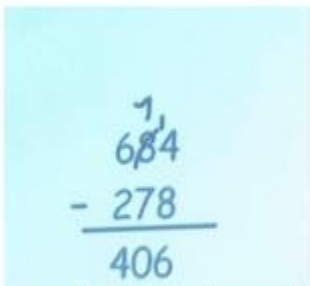
Both are very helpful concrete and pictorial representations of number but, crucially, they are representing different structures. The tens frame is accentuating and drawing attention to the '5 and a bit' structure of numbers, whereas Numicon draws attention to the odd/even structure. Both images support seeing the complement to 10 (i.e. what needs to be added to make 10).
The two images of 6, for example give different (equally important) ways of thinking about the structure of 6 which in turn influence that ways the children might transform, compare and combine numbers when calculating.

Big Ideas in Mastery: Mathematical Thinking

Messages
1. Mathematical thinking is central to deep and sustainable learning of mathematics.
2. Taught ideas that are understood deeply are not just 'received' passively but worked on by the learner. They need to be thought about, reasoned with and discussed.
3. Mathematical thinking involves:
   - looking for pattern in order to discern structure;
   - looking for relationships and connecting ideas;
   - reasoning logically, explaining, conjecturing and proving.

For example:
Asking "what's the same and what's different?" in a range of situations prompts and promotes mathematical thinking





Asking pupils to explain, convince, draw diagrams to illustrate an idea or strategy, reason and conjecture as a natural part of all activity in the mathematics classroom supports deep and sustainable learning.

Big Ideas of Mastery: Fluency

Messages

1. Fluency demands more of learners than memorisation of a single procedure or collection of facts. It encompasses a mixture of efficiency, accuracy and flexibility.
2. Quick and efficient recall of facts and procedures is important in order for learners' to keep track of sub problems, think strategically and solve problems.
3. Fluency also demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections and to make appropriate choices from a whole toolkit of methods, strategies and approaches.

For example:

Quick and accurate recall of all multiplication facts up to 12 × 12 is important in order to free working memory to see the big picture and make decisions about when to use this knowledge to solve certain problems.

However, if a pupil only knows these facts as an unconnected collection of memorised phrases and does not know:

- that 8 × 6 is the same as 6 × 8 or twice 4 × 6 or 12 less that 10 × 8;
  or
- does not know the connection between 6 × 8 and 16 × 8 or 6 × 80 or 0.6 × 8;
  or
- when faced with a problem of finding how many books are in a bookcase with 8 shelves and 6 books on each shelf, does not know what mathematics to use

… then they have not attained fluency.

Big Ideas in Mastery: Variation

Messages

1. The central idea of teaching with variation is to highlight the essential features of a concept or idea through varying the non-essential features.
2. When giving examples of a mathematical concept, it is useful to add variation to emphasise:
   a. What it is (as varied as possible);
   b. What it is not.
3. When constructing a set of activities / questions it is important to consider what connects the examples; what mathematical structures are being highlighted?
4. Variation is not the same as variety – careful attention needs to be paid to what aspects are being varied (and what is not being varied) and for what purpose.

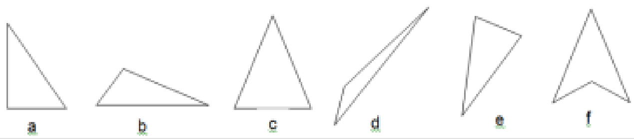
For example:

Procedural Variation:

58 – 24 = ___	36 – 25 = ___	53 – 22 = ___	49 – 24 = ___
57 – 25 = ___	46 – 24 = ___	64 – 23 = ___	48 – 25 = ___
56 – 26 = ___	56 – 23 = ___	75 – 24 = ___	47 – 26 = ___

Notice how the first and second numbers (the minuend and the subtrahend) in each column of calculations have been varied. This draws attention to the relationship between the two numbers in a subtraction and encourages some reasoning to explain why the answers change in the way they do.
Working on such questions can offer learners an opportunity for 'intelligent practice' where they can explain what is going on and make up their own examples.

Conceptual Variation:



To get a sense of what a triangle is learners need to see examples of triangles which show all aspects being varied (length of sides, angles, orientation). If most triangles are shown with one side as a horizontal base and the vertex pointing upwards (as in a, b and c), this feature might be over-generalised and pupils might think that d or e are not triangles.
It is also important to give non-examples, as in f and to discuss why this is not a triangle.